Links between GPDs and TMDs

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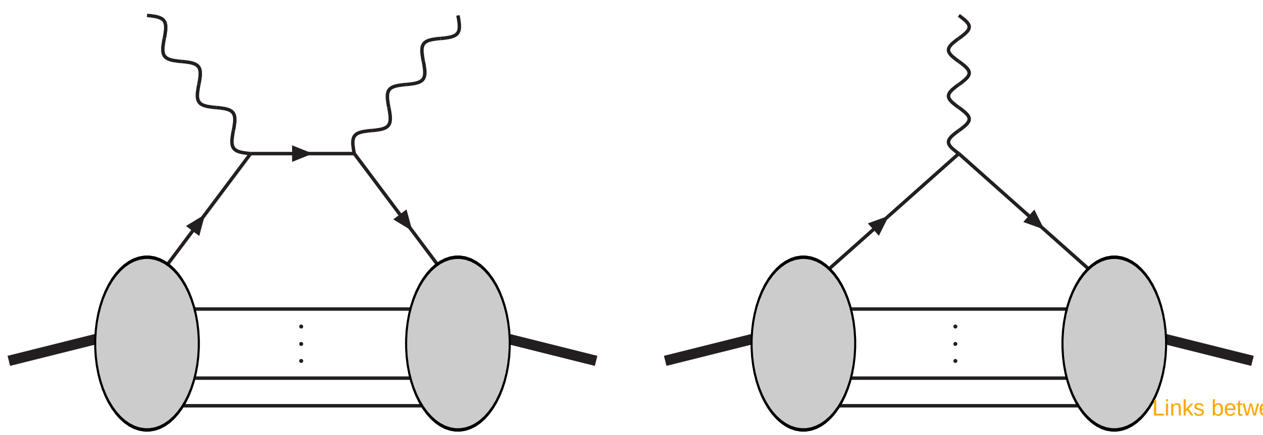
Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)



Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

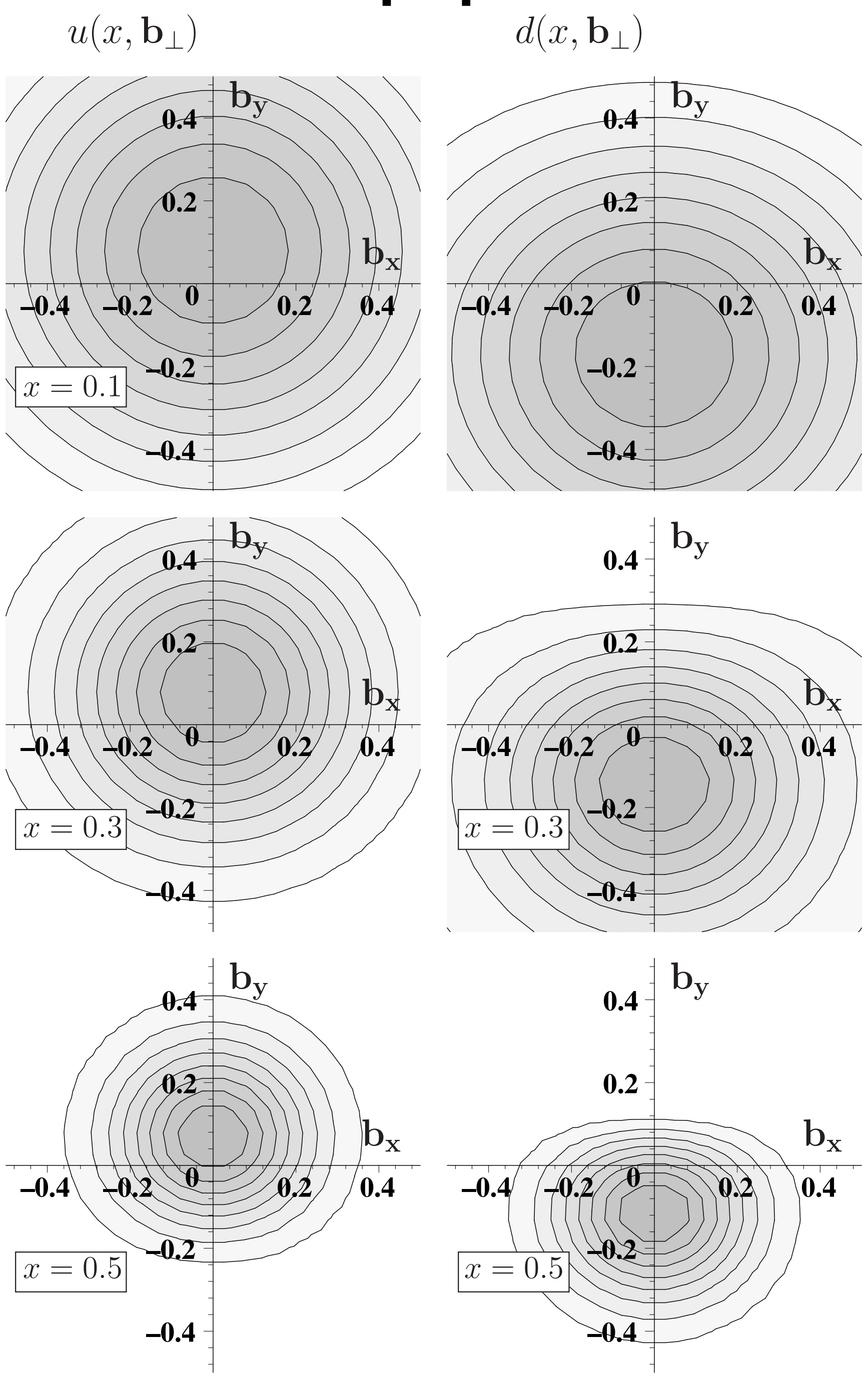
- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

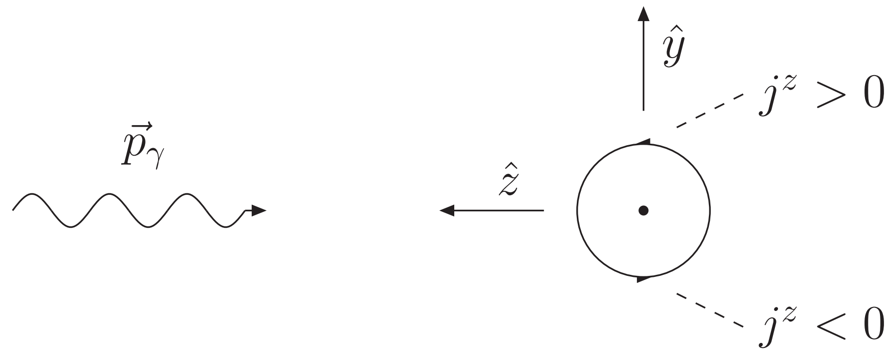
$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

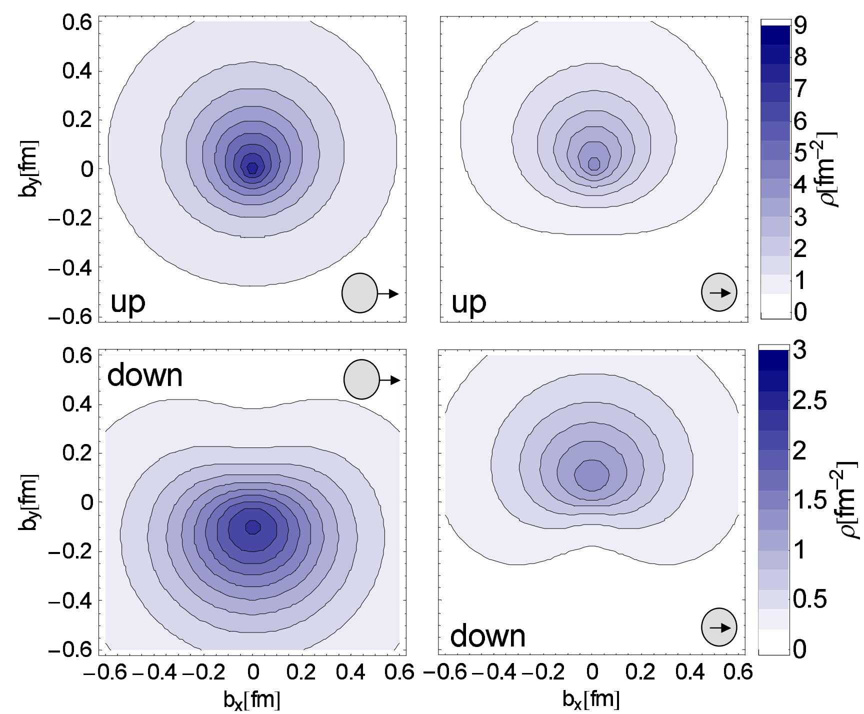
- DVCS amplitude

$$\mathcal{A}(\xi,t) \sim \int_{-1}^{1} \frac{dx}{x-\xi+i\varepsilon} GPD(x,\xi,t)$$

p polarized in $+\hat{x}$ direction



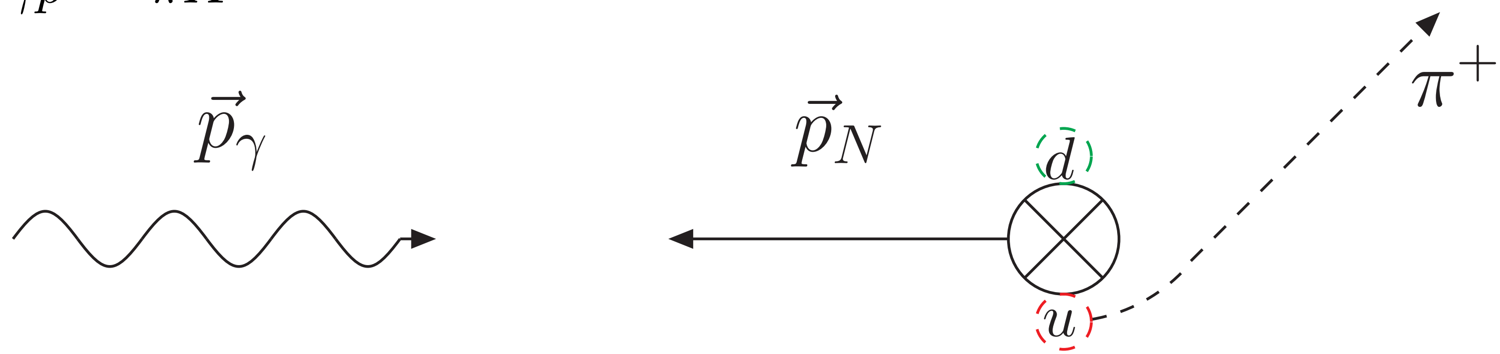




lattice results (Hägler et al.)

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$



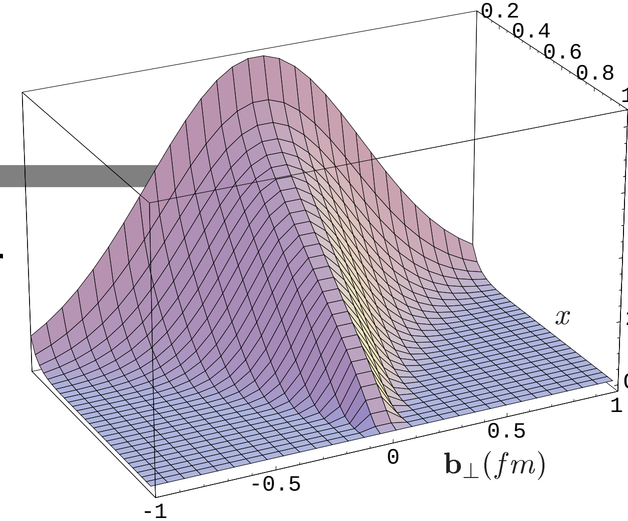
- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

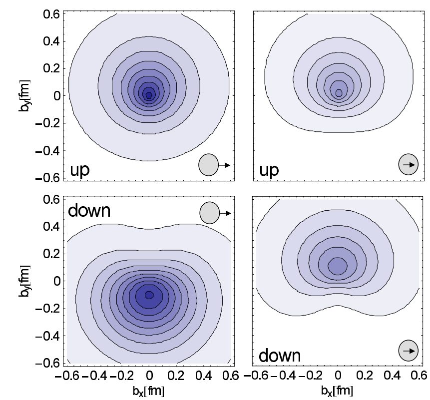
$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

Outline

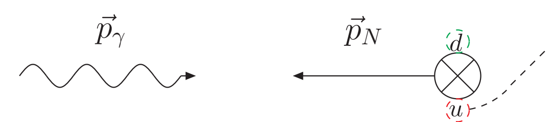


- Probabilistic interpretation of GPDs as Fourier trafos of impact parameter dependent PDFs
  - $H(x,0,-\boldsymbol{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\boldsymbol{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized



- Chromodynamik lensing and $\perp$ SSAs

$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\ \text{final state interactions}\end{array}\right\} \Rightarrow \perp \text{SSA in } \gamma N \longrightarrow \pi + X$$



- Transverse force on quarks in DIS
- Summary

Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist $\longrightarrow$ 'polarized quark distribution' $g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)$
- $\frac{1}{Q^2}$-corrections to X-section involve 'higher-twist' distribution functions, such as $g_2(x)$

$$\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- $g_2(x)$ involves quark-gluon correlations and does not have a parton interpretation as difference between number densities
- for $\perp$ polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- $\int dx\, x^2 g_T(x) \propto \langle PS|\bar{q}\gamma^\perp\gamma_5 D^+ D^+ \psi|PS\rangle$.
  - use Lorentz invariance and
  - equations of motion, e.g. $\gamma_\mu D^\mu q|PS\rangle = 0$
  - $\rightsquigarrow$ term involving $\int dx\, x^2 g_1(x)$ and term involving
  - $\langle PS|\bar{q}\gamma^+\gamma_5 \left[D^\perp, D^+\right] q|PS\rangle = \langle PS|\bar{q}\gamma^+\gamma_5 g G^{+\perp} q|PS\rangle$
- more generally: $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- $\sqrt{2} G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- sometimes called color-electric and magnetic polarizabilities $2M^2 \vec{S}\chi_E = \left\langle P,S \left| \vec{j}_a \times \vec{E}_a \right| P,S \right\rangle$ & $2M^2 \vec{S}\chi_B = \left\langle P,S \left| j_a^0 \vec{B}_a \right| P,S \right\rangle$ with $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- QED: $\bar{q}(0)eF^{+y}(0)\gamma^+ q(0)$ correlator between quark density $\bar{q}\gamma^+ q$ and ($\hat{y}$-component of the) Lorentz-force

$$F^y = e\left[\vec{E} + \vec{v} \times \vec{B}\right]^y = e\left(E^y - B^x\right) = -e\left(F^{0y} + F^{zy}\right) = -e\sqrt{2}F^{+y}.$$

for charged paricle moving with $\vec{v} = (0,0,-1)$ in the $-\hat{z}$ direction

$\hookrightarrow$ matrix element of $\bar{q}(0)eF^{+y}(0)\gamma^+ q(0)$ yields γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0,0,-1)$ would experience at that point

$\hookrightarrow$ d_2 a measure for the **color Lorentz force** acting on the struck quark in SIDIS in the instant **after being hit by the virtual photon**

$$\langle F^y(0) \rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

Quark-Gluon Correlations (Interpretation)

- Interpretation of d_2 with the transverse FSI force in DIS also consistent with $\langle k_\perp^y\rangle \equiv \int_0^1 dx \int \mathrm{d}^2k_\perp\, k_\perp^2 f_{1T}^\perp(x,k_\perp^2)$ in SIDIS (Qiu, Sterman)

$$\langle k_\perp^y\rangle = -\frac{1}{2p^+}\left\langle P,S \left| \bar{q}(0)\int_0^\infty dx^- gG^{+y}(x^-)\gamma^+ q(0)\right| P,S\right\rangle$$

  semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

- matrix element defining d_2 same as the integrand (for $x^- = 0$) in the QS-integral:
  - $\langle k_\perp^y\rangle = \int_0^\infty dt F^y(t)$ (use $\mathrm{d}x^- = \sqrt{2}\mathrm{d}t$)

  ↪ first integration point $\longrightarrow F^y(0)$

  ↪ (transverse) force at the begin of the trajectory, i.e. at the moment after absorbing the virtual photon

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$
  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S\left|\bar{q}(0)g\tilde{G}^{\mu\nu}(0)\gamma_\nu q(0)\right|P,S\right\rangle \sim f_2$

$\hookrightarrow$ different linear combination $f_2 = \chi_E - \chi_B$ of χ_E and χ_M

$\hookrightarrow$ combine with $d_2 \Rightarrow$ disentangle electric and magnetic force

- What should one expect (sign)?
  - $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$

  $\hookrightarrow$ d_2 positive/negative for u/d quarks in proton

  - large N_C: $d_2^{u/p} = -d_2^{d/p}$
  - consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$
- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$

$\hookrightarrow$ ($M^2 \approx 5\frac{\text{GeV}}{fm}$ $\quad \langle F_u^y(0)\rangle \approx -50\frac{\text{MeV}}{fm}$ $\quad \langle F_d^y(0)\rangle \approx 28\frac{\text{MeV}}{fm}$

- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Quark-Gluon Correlations (chirally odd)

- $\perp$ momentum for quark polarized in $+\hat{x}$-direction (unpolarized target)

$$\langle k_\perp^y \rangle = \frac{g}{2p^+} \left\langle P,S \left| \bar{q}(0) \int_0^\infty dx^- G^{+y}(x^-) \sigma^{+y} q(0) \right| P,S \right\rangle$$

- compare: interaction-dependent twist-3 piece of $e(x)$

$$\int dx x^2 e^{int}(x) \equiv e_2 = \frac{g}{4MP^{+2}} \langle P,S \left| \bar{q}(0) G^{+y}(0) \sigma^{+y} q(0) \right| P,S \rangle$$

$\hookrightarrow$ $\langle F^y \rangle = M^2 e_2$

$\hookrightarrow$ (chromodynamic lensing) $e_2 < 0$

Summary

- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\hookrightarrow$ $\kappa^{q/p} \Rightarrow$ sign of deformation

$\hookrightarrow$ attractive FSI $\Rightarrow f_{1T}^{\perp u} < 0$ & $f_{1T}^{\perp d} > 0$

- Interpretation of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as $\perp$ **force on active quark in DIS in the instant after being struck by the virtual photon**

$$\langle F^y(0)\rangle = -M^2 d_2 \qquad\qquad \text{(rest frame; } S^x = 1\text{)}$$

- In combination with measurements of f_2
  - color-electric/magnetic force $\frac{M^2}{4}\chi_E$ and $\frac{M^2}{2}\chi_M$
- $\kappa^{q/p} \Rightarrow \perp$ deformation $\Rightarrow d_2^{u/p} > 0$ & $d_2^{d/p} < 0$ (attractive FSI)
- combine measurement of d_2 with that of $f_{1T}^\perp \Rightarrow$ **range of FSI**
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + E_T^q$

$\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks

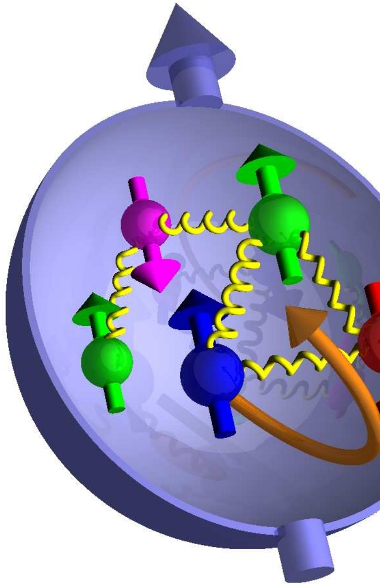
$\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect:

$$h_1^{\perp,q} < 0 \qquad\qquad |h_1^{\perp,q}| > |f_{1T}^q|$$

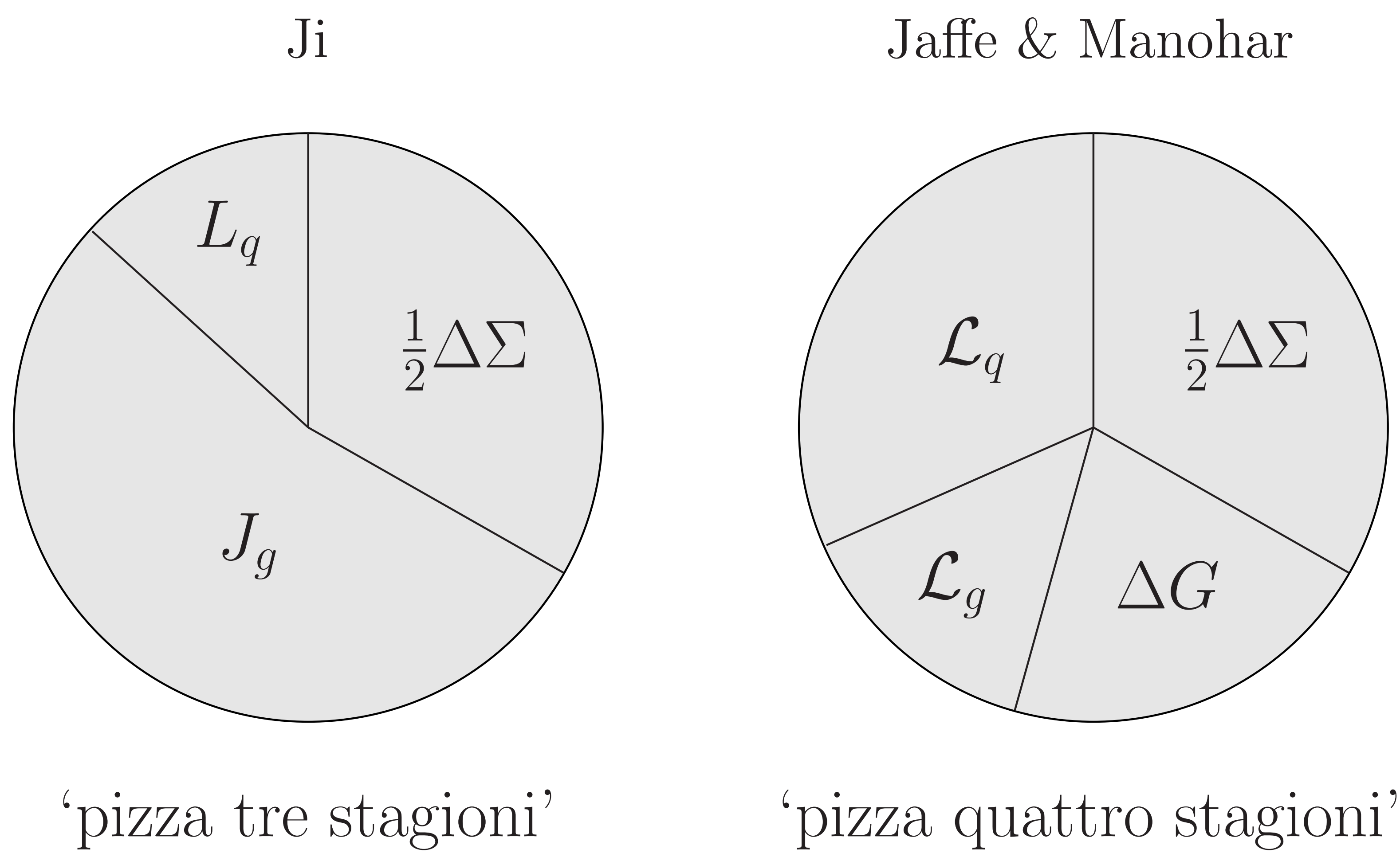
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\longrightarrow$ Boer-Mulders)

What is Orbital Angular Momentum?



- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe

The nucleon spin pizza(s)



- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

Angular Momentum Operator

- angular momentum tensor $M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho}$
- $\partial_\rho M^{\mu\nu\rho} = 0$

$\hookrightarrow$ $\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk}\int d^3r M^{jk0}$ conserved

$$\frac{d}{dt}\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x\partial_0 M^{jk0} = \frac{1}{2}\varepsilon^{ijk}\int d^3x\partial_l M^{jkl} = 0$$

- $M^{\mu\nu\rho}$ contains time derivatives (since $T^{\mu\nu}$ does)
  - use eq. of motion to get rid of these (as in T^{0i})
  - integrate total derivatives appearing in T^{0i} by parts
  - yields terms where derivative acts on x^i which then 'disappears'

  $\hookrightarrow$ J^i usally contains both
    - 'Extrinsic' terms, which have the structure '$\vec{x}\times$ Operator', and can be identified with 'OAM'
    - 'Intrinsic' terms, where the factor $\vec{x}\times$ does not appear, and can be identified with 'spin'

Angular Momentum in QCD (Ji)

- following this general procedure, one finds in QCD

$$\vec{J} = \int d^3x \left[\psi^\dagger \vec{\Sigma} \psi + \psi^\dagger \vec{x} \times \left(i\vec{\partial} - g\vec{A} \right) \psi + \vec{x} \times \left(\vec{E} \times \vec{B} \right) \right]$$

  with $\Sigma^i = \frac{i}{2}\varepsilon^{ijk}\gamma^j\gamma^k$

- Ji does not integrate gluon term by parts, nor identify gluon spin/OAM separately
- Ji-decomposition valid for all three components of $\vec{J}$, but usually only applied to $\hat{z}$ component, where the quark spin term has a partonic interpretation

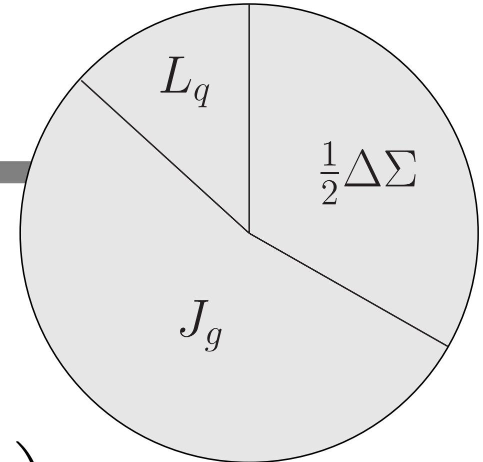
(+) all three terms manifestly gauge invariant

(+) DVCS can be used to probe $\vec{J}_q = \vec{S}_q + \vec{L}_q$

(-) quark OAM contains interactions

(-) only quark spin has partonic interpretation as a single particle density

Ji-decomposition



- Ji (1997)

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

with ($P^\mu = (M,0,0,1)$, $S^\mu = (0,0,0,1)$)

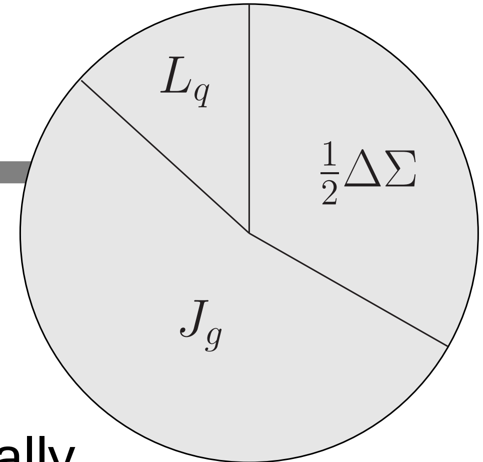
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x})\, |P,S\rangle \qquad \Sigma^3 = i\gamma^1\gamma^2$$

$$L_q = \int d^3x \, \langle P,S|\, q^\dagger(\vec{x}) \left(\vec{x}\times i\vec{D}\right)^3 q(\vec{x})\, |P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D}\right) q + \vec{r} \times \left(\vec{E} \times \vec{B}\right)$ applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities
- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix elements of $q^\dagger \left(\vec{r} \times i\vec{D}\right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) that is local and manifestly gauge invariant has not been found

L_q for proton from Ji-relation (lattice)

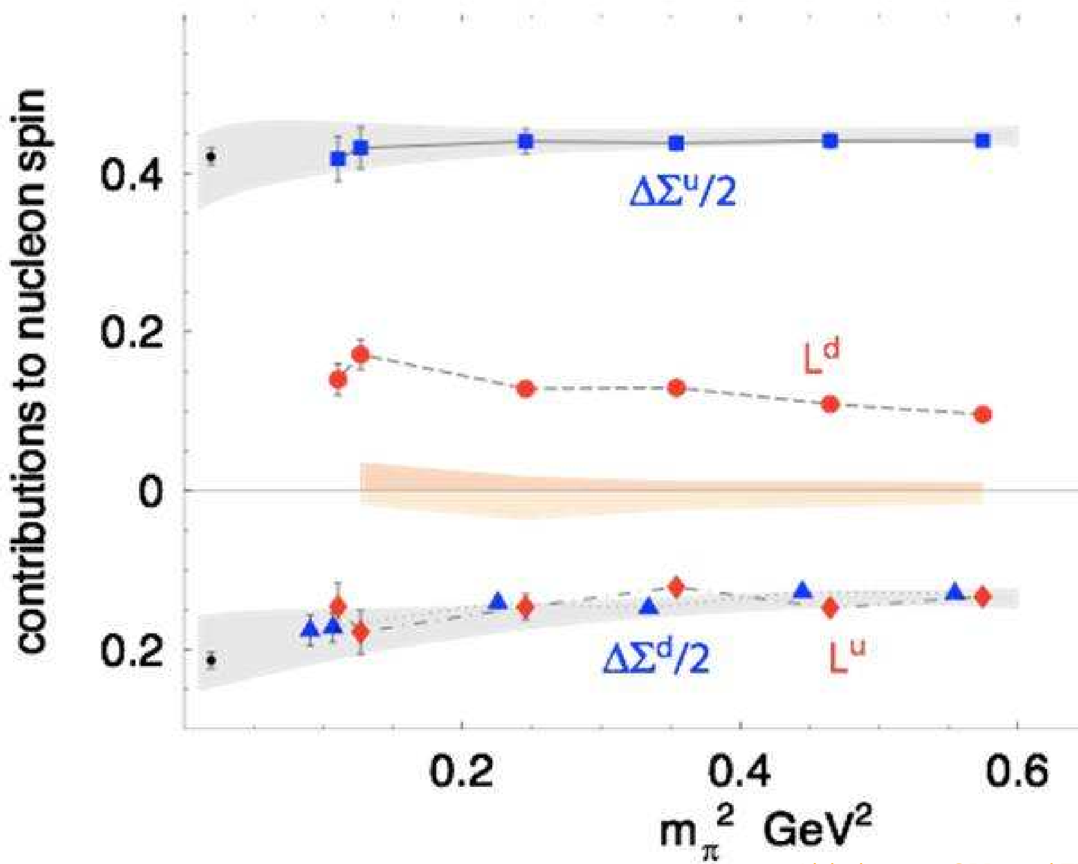
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0) + E_q(x,0) \right] x.$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypesurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2x_\perp \int dx^- M^{12+}$$

where $x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^-\right)$ and $M^{12+} = \frac{1}{\sqrt{2}}\left(M^{120} + M^{123}\right)$

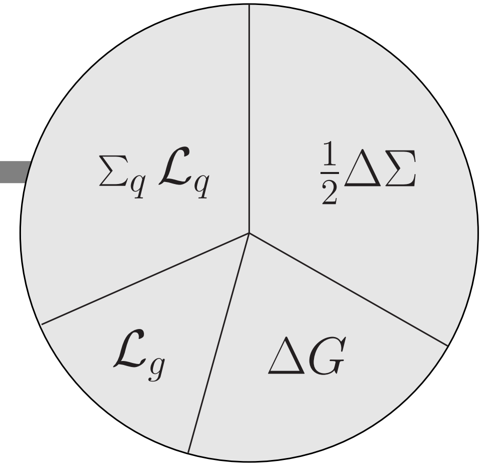
- Since $\partial_\mu M^{12\mu} = 0$

$$\int d^2\mathbf{x}_\perp \int dx^- M^{12+} = \int d^2\mathbf{x}_\perp \int dx^3 M^{120}$$

(compare electrodynamics: $\vec{\nabla}\cdot\vec{B} = 0 \quad \Rightarrow$ flux in = flux out)

- use eqs. of motion to get rid of ‘time’ (∂_+ derivatives) & integrate by parts whenever a total derivative appears in the T^{i+} part of M^{12+}

Jaffe/Manohar decomposition



- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

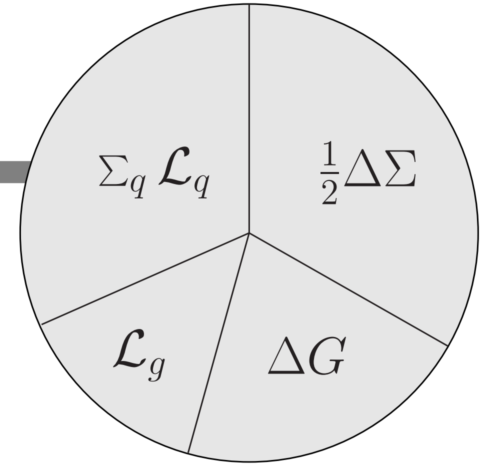
where $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\mathcal{L}_q = \int d^3r \, \langle P,S | \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S | \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2 \int d^3r \, \langle P,S | \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle$$

Jaffe/Manohar decomposition



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\vec{p}\,\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to 'mix' Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q = \bar{q}\gamma^0\left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+\left[\vec{r}\times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- For nucleon at rest, matrix element of L_q same as that of $\bar{q}\gamma^+\left[\vec{r}\times\left(i\vec{\partial}-g\vec{A}\right)\right]^z q$

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger\left(\vec{r}\times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger\left(xgA^y - ygA^x\right)q\Big|_{A^+=0}$

Summary part 1:

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\vec{p}\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[M.B. + Hikmat Budhathoki Chhetri (BC), PRD **79**, 071501 (2009)]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

↪ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x,\mathbf{k}_\perp) = \left(M + \frac{m}{x}\right)\phi \qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1 + ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

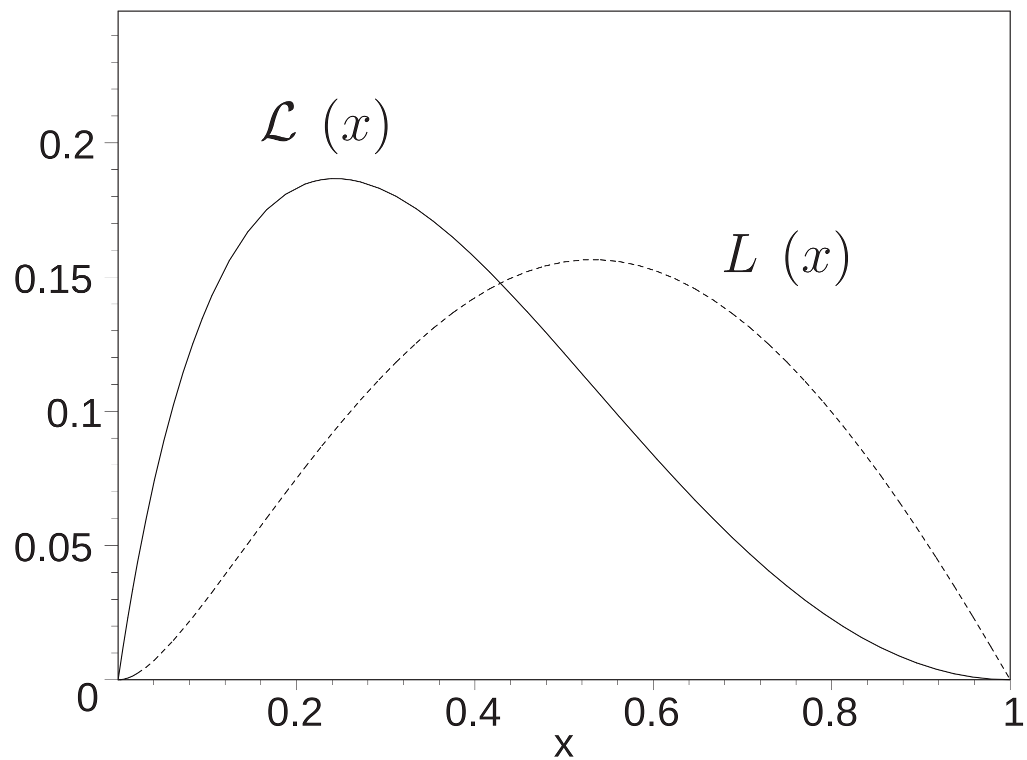
⇝ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is <u>not</u> a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^\uparrow_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ ‘unintegrated Ji-relation’ does not yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1 + ik^2}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$$

- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

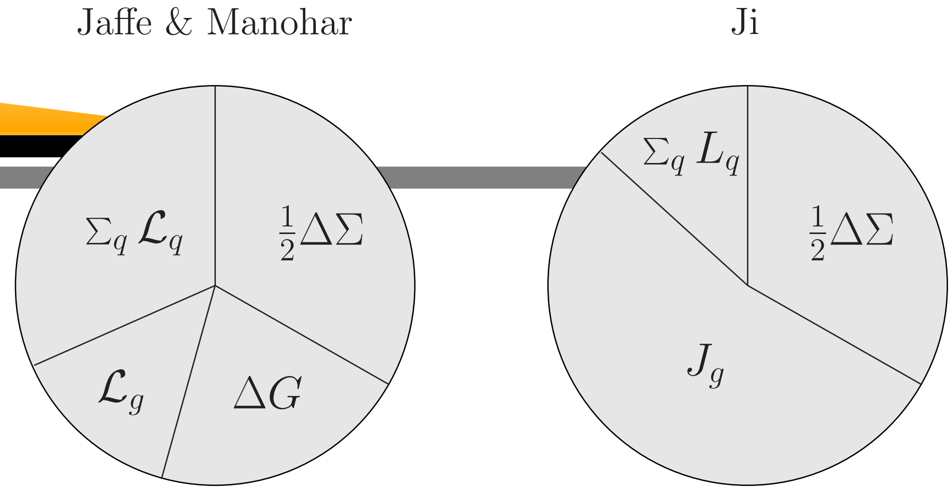
- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and defining $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields $\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$

- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- ↪ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- ↪ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d > L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- ↪ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary



- inclusive $\vec{e}\,\overleftarrow{p}$ / $\vec{p}\,\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does <u>not</u> yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for O (α_s) dressed quark

Announcement:

- workshop on Orbital Angular Momentum of Partons in Hadrons
- ECT* 9-13 November 2009
- organizers: M.B. & Gunar Schnell
- confirmed participants: M.Anselmino, H.Avakian, A.Bacchetta, L.Bland, D.Fields, L.Gamberg, G.Goldstein, M.Grosse-Perdekamp, P.Hägler, X.Ji, R.Kaiser, E.Leader, S.Liutti, N.Makins, A.Miller, D.Müller, P.Mulders, A.Schäfer, G.Schierholz, O.Teryaev, W.Vogelsang, F.Yuan